Printed Pages:03 **Sub Code: KCS -303**

Paper Id: 233534 **Roll No.** | | | | | | | | | | |

B. TECH.
(SEM-III) THEORY EXAMINATION 2022-23
DISCRETE STRUCTURES & THEORY OF LOGIC

Time: 3 Hours ***Total Marks: 100***

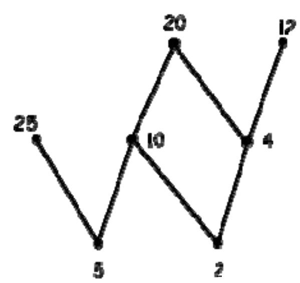
Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

(a) Identify whether $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$, $\forall x,y \in R$, where $\lceil x \rceil$ is a ceiling function

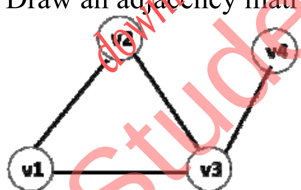
(b) Find the Maximal elements and minimal elements form the following Hasse's diagram



(c) Define what is Big-O notation with respect of growth of functions.

(d) Find the composite mapping **gof** if
f: R→R is given by $\mathbf{f(x) = e^x}$ and g: R→R is given by $\mathbf{g(x) = \sin x}$

(e) Draw an adjacency matrix for the following graph



(f) Let $\mathbf{A = \{ \Phi, b\}}$, then calculate $\mathbf{A \cup P(A)}$, where P(A) is a power set of A.

(g) Draw the Hasse's diagram of the POSET $(L, \subseteq)$, where
$L = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$, where the sets are given by
$S_0 = \{a,b,c,d,e,f\}$, $S_1 = \{a,b,c,d,e\}$, $S_2 = \{a,b,c,e,f\}$,
$S_3 = \{a,b,c,e\}$, $S_4 = \{a,b,c\}$, $S_5 = \{a,b\}$, $S_6 = \{a,c\}$, $S_7 = \{a\}$

(h) Describe Planar graph and express Euler's formula for planar graph.

(i) Define normal subgroup.

(j) Identify whether $\mathbf{(p \land q) \rightarrow (p \lor q)}$ is tautology or contradiction with using Truth table.

SECTION B

2. Attempt any *three* of the following: 10x3=30

(a) Identify whether the each of the following relations defined on the set X = {1,2,3,4} are reflexive, symmetric, transitive and/or antisymmetric?

(i) R_1 = { (1,1), (1,2), (2,1) }

(ii) R_2 = { (1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4) }

(iii) R_3 = { (2,1), (3,1), (3,2), (4,1), (4,2), (4,3) }

(b) Let a function is defined as **f: R-{3}→ R-{1}, f(x) = (x-1)/(x-3)** , then **show** that f is a bijective function and also compute the inverse of f. Where R is a set of real numbers.

(c) **(i)** Express Converse, Inverse and Contrapositive of the following statement **"If x+5=8 then x=3"**
(ii) Show that the statements **P↔Q** and **(P ∧ Q) V(⌉ P ∧ ⌉ Q)** are equivalent

(d) Express the following
- (i) Euler graph and Hamiltonian graph
- (ii) Chromatic number of a graph
- (iii) Walk and path
- (iv) Bipartite graph

(e) Solve the following recurrence relation by using **generating function.**
$a_n + 5a_{n-1} + 6a_{n-2} = 42 \cdot 4^n$ **, where** $a_0 = 1$ **and** $a_1 = -2$

SECTION C

3. Attempt any *one* part of the following: 10x1=10

(a) Let **G = {1,-1,** *i, i* } with the operation of ordinary multiplication on G be an algebraic structure, where $i=\sqrt{-1}$.
- (i) Determine whether G is abelian.
- (ii) Determine the order of each element in G.
- (iii) Determine whether G is a cyclic group, if G is a cyclic group, then determine the generator/generators of the group G.
- (iv) Determine a subgroup of the group G.

(b) Let (G,*) and G',*') be any two groups and let e and e' be their respective identities. If f is a homomorphism of G into G', then prove that
- (i) f(e) = e'
- (ii) $f(x^{-1}) = [f(x)]^{-1}$, ∀ x ∈G

4. Attempt any *one* part of the following: 10x1=10

(a) Use generating function to find the number of ways Rs 23 can by paid by using 4 coins of Rs 5, 6 coins of Rs 2 and 4 coins of Rs 1.

(b) Using Pigeonhole principle find the minimum number n of integers to be selected from S={1,2,3,4,5,6,7,8,9} so that
(i) the sum of two of the integers is even
(ii) the difference of two of the n integers is 5

5. Attempt any *one* part of the following: 10x1=10

(a) Define complemented lattice and then show that in a distributive lattice, if an element has a complement then this complement is unique.

(b) Solve the following Boolean functions using K-map:
(i) $F(A,B,C,D) = \sum(m_0, m_1, m_2, m_4, m_5, m_6, m_8, m_9, m_{12}, m_{13}, m_{14})$

(ii) **F(A,B,C,D)=∑(0,2,5,7,8,10,13,15)**

6. Attempt any *one* part of the following: 10x1=10

(a) Prove the validity of the following argument.
If Mary runs for office, She will be elected. If Mary attends the meeting, she will run for office. Either Mary will attend the meeting or she will go to India. But Mary cannot go to India.
"Thus Mary will be elected".

(b) Convert the following two statements in quantified expressions of predicate logic

(i) For every number there is a number greater than that number.
(ii) Sum of every two integer is an integer.
(iii) Not Every man is perfect.
(iv) There is no student in the class who knows Spanish and German

7. Attempt any *one* part of the following: 10x1=10

(a) Prove that the set of residues F={0,1,2,3,4} modulo 5 is a field w.r.t. addition and multiplication of residue classes modulo 5. i.e. (F, $+_5$, X_5) is a field.

(b) Define Boolean algebra. **Show** that **a'.[(b'+c)'+ b.c] + [(a+b')'.c] = a'.b** using rules of Boolean Algebra. Where **a'** is the complement of an element **a**.